Pergamon

Int. J. Multiphase Flow Vol. 22, No. 3, pp. 515–525, 1996

S0301-9322(96)00005-5
0301-9322/96 $15.00 + 0.00

THE PARTICLE-IN-A-TUBE ANALOGY FOR A MULTIPARTICLE SUSPENSION

R. DI FELICE

Istituto di Ingegneria Chimica e di Processo "G.B. Bonino", Università degli Studi, via Opera Pia, 15, 16145 Genova, Italy

(Received 30 May 1995; in revised form 30 January 1996)

Abstract—A simple analogy between a multiparticle suspension and a single particle in a tube has been obtained by using the concept of "hydraulic diameter". Fully theoretical derivations for the effect of the wall on the single particle enable the solid–fluid interaction force to be estimated with no empirical input for the viscous and the inertial flow regimes. The analogy is successfully tested in the intermediate flow regime and finally the expansion characteristics of homogeneous fluidised beds is obtained, in good agreement with the Richardson & Zaki (1954) equation, by using experimental data on the terminal settling velocity of a lone particle in a tube. Copyright © 1996 Elsevier Science Ltd.

Key Words: multiparticle systems, drag force, voidage function, wall function

1. INTRODUCTION

The fluid dynamic description of multiparticle solid–fluid systems presents unresolvable difficulties when the subject is tackled from a purely theoretical point of view: the effect of the presence of neighbouring particles on a specific particle has been quantified theoretically only in the viscous flow regime, and there only for limited cases as for example fixed spatial arrangement of particles (Happel 1958), or random dilute suspensions (Batchelor 1972).

In contrast, semi-empirical or empirical approaches have flourished, in which the particle–particle fluid dynamic interaction is represented in a simplified yet satisfactory manner: typical examples are the pseudo-fluid model and the analogy with the flow in straight pipes. In the pseudo-fluid approach the effect of the surrounding particles on the drag force on a given particle is accounted by modifying, somewhat arbitrarily, the fluid density and viscosity; on the other hand in the analogy with flow in pipes the hydrodynamic fluid–particle interaction in multiparticle systems is inferred from the knowledge of the tube wall–fluid interaction in empty pipes, on the basis of a geometrical analogy. These approaches have been recently reviewed and discussed at length (Di Felice 1995).

The analogy with flow in pipes has been particularly popular for fluid flow in porous media, the Carman–Kozeny and Burke–Plummer equations for the pressure drop in fixed beds, for the viscous and inertial regimes respectively being striking examples (Bird *et al.* 1960). In this work a similar operation is attempted by considering the fluid dynamic analogy of a particle in a multisphere suspension and a lone particle in a cylindrical tube. The influence of the surrounding spheres on the drag force acting on a test sphere is estimated from the knowledge of the effect of the tube wall on the drag force acting on a single sphere.

The tube wall and the particle suspension effect the hydrodynamics in a similar way: they restrict the area available for fluid flow and, as a result, increase the drag force on the test particle. The effective distance of the surrounding particles from the test particle (quantified by the voidage fraction, ϵ) corresponds to the distance from the wall to the single particle in the tube (quantified by the ratio of the particle to tube diameter $d/D = \lambda$).

The potential advantage of this approach lies in the fact that a single sphere in a tube is more easily treated theoretically than the corresponding solid–fluid suspension. This is because in such system only one fluid dynamic interaction, namely the particle–wall interaction, need be considered. On the other hand, the particle in a suspension interacts with all the other particles, so that a large number of interactions have to be taken into account in that case. The same advantage applies even to experimental studies as working with one particle is certainly easier than working with many.

This idea is by no means new. At the end of last century Munroe (1888) obtained, from an intuitive reasoning, that today would be regarded as cell model approach, the following equivalence

$$\lambda = (1 - \epsilon)^{1/3}. \qquad [1]$$

He then went on to measure experimentally the hindering effect of the wall on a single sphere as a function of λ, and was thereby able to estimate the minimum fluidisation velocity of the equivalent solid–liquid system by means of [1] in satisfactory agreement with measured values.

Munroe's work related to systems in the inertial flow regime, whereas here we will consider the whole Reynolds number range; the analysis will still be limited, however, to spherical particles and cylindrical tubes.

2. THE PARTICLE-IN-A-TUBE ANALOGY

The geometrical analogy

We can start by considering a homogeneous suspension of solid spheres: the well known analogy with flow in a cylindrical wall tube considers the fluid to be moving, relative to the wall, with velocity u_c equal to the actual fluid–solid relative velocity in the suspension, u_0/ϵ (u_0 being the fluid superficial velocity when the solid are at rest). The equivalent tube diameter (the 'hydraulic diameter') is given by:

$$D_h = \frac{2\epsilon d}{3(1 - \epsilon)}. \qquad [2]$$

This equivalence is illustrated in figure 1.

The next step is to move from the system of figure 1 to the case we consider here, depicted in figure 2: this involves inserting in both the empty tube and in the suspension a foreign sphere identical to those making up the multiparticle system. The suspension has to rearrange itself in order to accommodate the new sphere; it does so in order to preserve the same average properties, including hydraulic diameter, [2], as before the insertion: nothing changes. But for the tube we must consider the change in the hydraulic diameter brought about by the included particle. The dominant effect of drag on the particle in the tube will be felt close to its horizontal circumference where the fluid velocity is a maximum; the hydraulic diameter at this plane, equated to the hydraulic

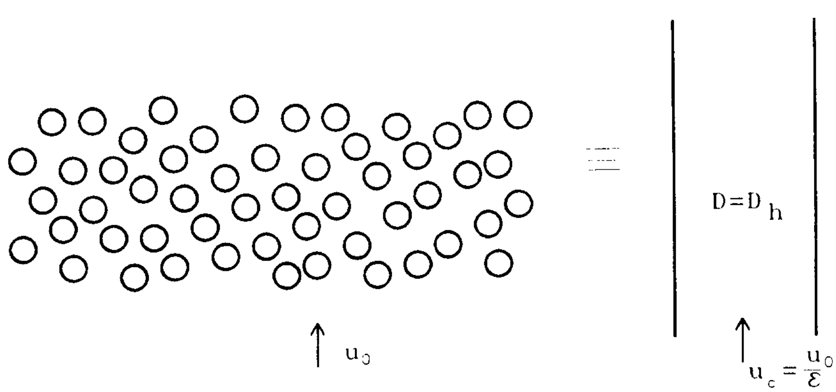

Figure 1. The analogy between a multiparticle suspension and an empty tube.

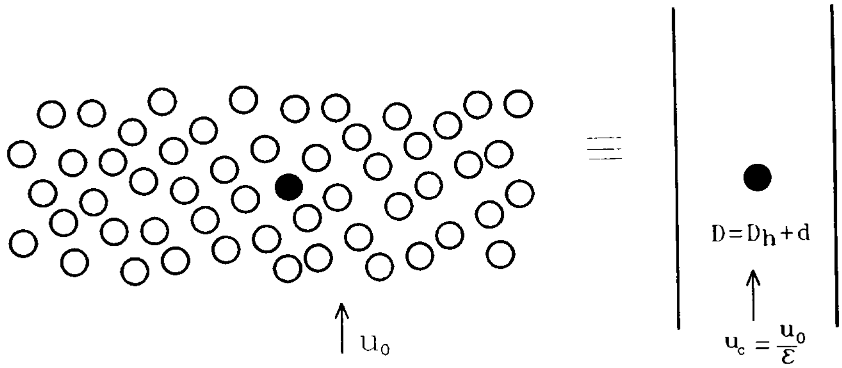

Figure 2. The analogy between a multiparticle suspension and a single particle in a tube.

diameter of the suspension D_h, leads to

$$D = D_h + d \qquad [3]$$

and the ratio λ for the systems in figure 2 is then obtained by [2] and [3]:

$$\lambda = \frac{3(1-\epsilon)}{3-\epsilon}. \qquad [4]$$

Equation [4] is the relationship defining the geometrical analogy of a single particle in a tube with a multiparticle suspension: it is shown graphically in figure 3.

As in the original analogy, we will assume that the fluid in the tube will approach the particle with a velocity $u_c = u_0/\epsilon$.

The wall function and the voidage function

A single particle in an infinite expanse of fluid will experience a drag force given by

$$F_{D0} = C_{D0}\frac{\rho u_0^2}{2}\frac{\pi d^2}{4} \qquad [5]$$

where the drag coefficient, C_{D0}, is a well known empirical function of the particle Reynolds number,

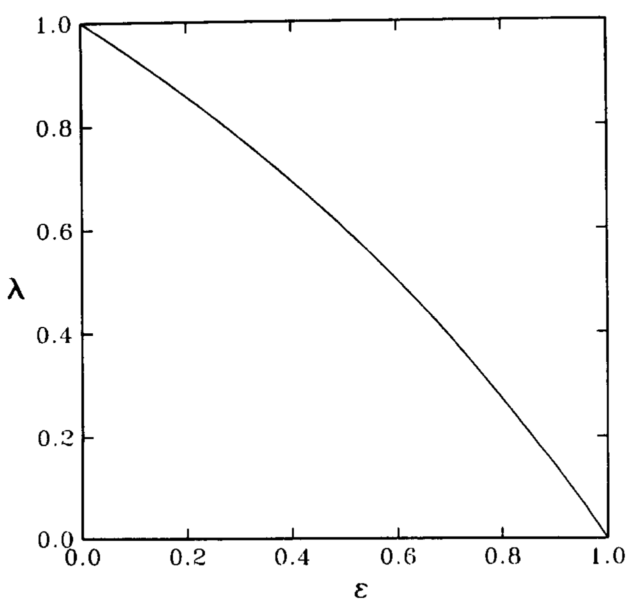

Figure 3. The geometrical relationship between ϵ and λ.

Table 1.

System	Approaching velocity of the fluid relative to the particle, u	Reynolds number $d\rho u/\mu$	Drag force $kC_D\mathrm{Re}^2$	Drag coefficient $C_D = f(\mathrm{Re})$
Single particle in an infinite expanse of fluid	u_0	Re_0	F_{D0}	C_{D0}
Particle in a multiparticle suspension	u_0	Re_0	F_D	$g(\epsilon)C_{D0}$
Single particle in a very large tube	u_c†	Re_c	F_{Dc}	C_{Dc}
Single particle in a finite size tube	u_c†	Re_c	F_D	$f(\lambda)C_{Dc}$
Single particle in an infinite expanse of fluid in terminal condition	$u_{t\infty}$	$\mathrm{Re}_{t\infty}$	$F_{Dt\infty}$	$C_{Dt\infty}$
Single particle in a tube in terminal condition	u_t	Re_t	F_D	$f(\lambda)C_{Dt}$

†Equal to u_0/ϵ in the present analogy.

$\mathrm{Re}_0 = d\rho u_0/\mu$, as for example given by Dallavalle (1948),

$$C_{D0} = \left(0.63 + \frac{4.8}{\mathrm{Re}_0^{0.5}}\right)^2. \quad [6]$$

(The reader is referred to table 1 for the meaning of the different symbols for velocity, Reynolds number, drag force and drag coefficient used in this paper.)

For a given system, [5] can be written as a function of the Reynolds number alone

$$F_{D0} = kC_{D0}\mathrm{Re}_0^2 \quad [7]$$

where the coefficient k is given by

$$k = \frac{\pi\mu^2}{8\rho}. \quad [8]$$

The presence of the other particles in the suspension will effect the drag force. When the superficial velocity is kept constant, this effect is conveniently quantified by a *voidage function* $g(\epsilon)$ by which the drag force on a single particle is to be multiplied in order to obtain the drag force on the same particle in a suspension

$$F_D = g(\epsilon)F_{D0} = g(\epsilon)kC_{D0}\mathrm{Re}_0^2. \quad [9]$$

Of course, we would like to obtain theoretical expressions for $g(\epsilon)$; in practice, as said in the introduction, this is only possible for limited cases and $g(\epsilon)$ has been evaluated numerically from experimental data on fixed and suspended particle systems over the full practical range of flow regime and particle concentration (Di Felice 1994).

We can reason in an analogous way for the quantification of the wall effect on the drag on a single sphere in a bounded medium (in this case, for the analogy, the fluid velocity is $u_c = u_0/\epsilon$). When the wall effect is negligible ($\lambda \to 0$) then, as in [7],

$$F_{Dc} = kC_{Dc}\mathrm{Re}_c^2 \quad [10]$$

where $\mathrm{Re}_c = d\rho u_c/\mu$, and C_{Dc} is the corresponding drag coefficient obtained as in [6]

$$C_{Dc} = \left(0.63 + \frac{4.8}{\mathrm{Re}_c^{0.5}}\right)^2 = \left(0.63 + \frac{4.8\epsilon^{0.5}}{\mathrm{Re}_0^{0.5}}\right)^2. \quad [11]$$

The wall will increase the drag on the particle by a factor $f(\lambda)$:

$$F_D = f(\lambda)F_{Dc} = f(\lambda)kC_{Dc}\mathrm{Re}_c^2 \quad [12]$$

which, in analogy with the voidage function, we call *wall function*, although previous workers have used slightly different names such as "wall correction factor" (Haberman & Sayre 1958) or "drag factor" (Clift *et al.* 1978). It must be stressed that the wall function is defined for a constant fluid approach velocity relative to the particle, as was for the case of the voidage function.

If we now suppose that the effect of the wall on the test sphere is equivalent to the effect of the neighbouring suspension spheres, then the two expressions for this drag force, [7] and [12], must be equivalent:

$$F_D = g(\epsilon)kC_{D0}^2\mathrm{Re}_0 = f(\lambda)kC_{Dc}^2\mathrm{Re}_c \tag{13}$$

which, by taking into account the relation between Re_0 and Re_c, simplifies to

$$g(\epsilon)C_{D0}\epsilon^2 = f(\lambda)C_{Dc}. \tag{14}$$

Equation [14], coupled with the geometrical similarity [4], then provides the full analogy.

If we know $f(\lambda)$ then we can obtain $g(\epsilon)$. In order to test this relationship, we use it to estimate the voidage function from theoretical expression of the wall function. For cases where theoretical analyses are not available, we make do with experimental findings. The voidage functions so derived are all compared with corresponding empirical expressions: it has been demonstrated that for a wide variety of both fixed-bed and suspended-particle systems, the voidage function could be expressed as

$$g(\epsilon) = \epsilon^{-\beta} \tag{15}$$

where the exponent β is dependent in the particle Reynolds number as illustrated in figure 4 (Di Felice 1994).

3. VALIDATION OF THE ANALOGY

The low Re regime

For this fluid dynamic regime [14] reduces to

$$g(\epsilon) = \frac{f(\lambda)}{\epsilon}. \tag{16}$$

The effect of the wall on the drag force on a sphere in the viscous flow regime has been extensively studied. Happel & Brenner (1973) have reported the most important investigations; of these the one most relevant is probably that based on the work of Haberman & Sayre (1958) where $f(\lambda)$ was derived for the whole range of λ. The results are presented both in analytical form, the

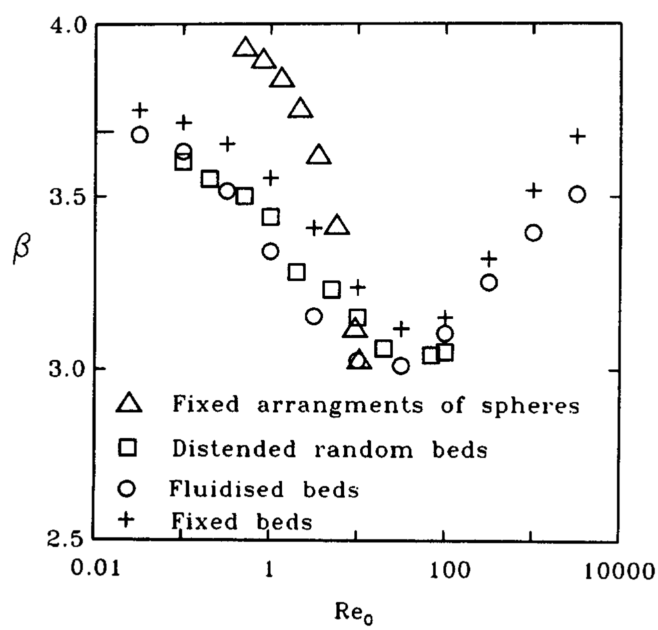

Figure 4. The exponent β evaluated from a variety of multiparticle–fluid systems.

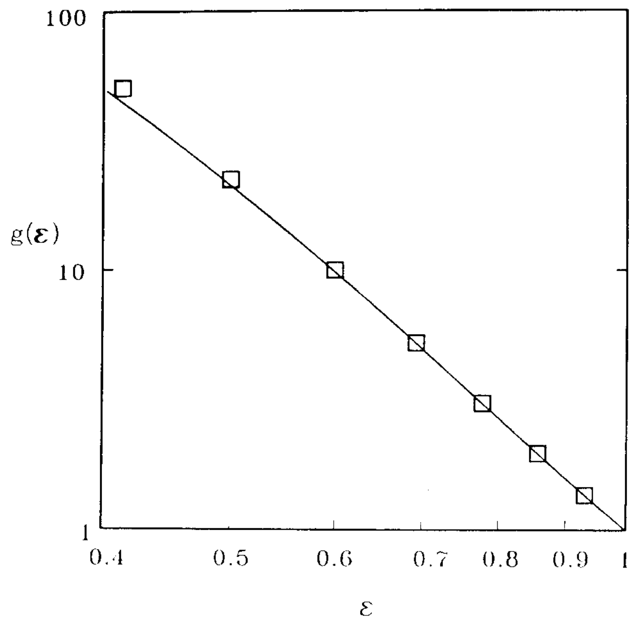

Figure 5. The voidage function evaluated with the present analogy in the viscous flow regime. Continuous line obtained from [17], points from tabled data of Paine & Scherr (1975).

following relationship being valid for λ values up to 0.6

$$f(\lambda) = \frac{1 - 0.75857\lambda^5}{1 - 2.1050\lambda + 2.0865\lambda^3 - 1.7068\lambda^5 + 0.72603\lambda^6} \qquad [17]$$

and numerically for values of λ up to 0.90 (Paine & Scherr 1975).

Equation [16], together with the relationship between λ and ϵ [4], lead to estimates for $g(\epsilon)$. This function is plotted, in logarithm coordinates, in figure 5, as a function of ϵ. The figure shows a practically linear relationship, indicating that $g(\epsilon)$ can be represented by [15]. In the specific case evaluated here, the parameter β is approximately 4.3. This finding compares well with experimental evaluations of the voidage function for multiparticle suspensions in the viscous flow regime. From figure 4 it is evident that β is around 3.7–4.

The high Re regime

The relation between wall and voidage function is, in this case,

$$g(\epsilon) = \frac{f(\lambda)}{\epsilon^2}. \qquad [18]$$

The theoretical derivation of the wall function in this regime is due to Newton (1687):

$$f(\lambda) = \frac{1}{(1 - \lambda^2)^2(1 - 0.5\lambda^2)}. \qquad [19]$$

The validity of [19] has been recently verified for experimental conditions ranging over the whole spectrum of λ (Di Felice *et al.* 1995).

The voidage function is easily derived and it is plotted, in log coordinates, in figure 6. Again the voidage function can be approximate with little error by a straight line, the average slope β being in this case close to 3.7, in excellent agreement with experimental findings for multiparticle suspensions.

The intermediate flow regime

Perhaps surprisingly, no theoretical derivation exists for the determination of the wall function in the intermediate flow regime. This lack of theoretical information leaves only the option of confronting the analogy with experimental data.

The most extensive work on the subject is certainly that of Fidleris & Whitmore (1961). They measured the retarding effect of the wall on the particle terminal velocity for the whole range of flow regime, and presented their results only in a graphical form.

For a particle in a tube under terminal conditions, $u_c = u_t$, the drag force is given by

$$F_D = F_{Dt\infty} = kC_{Dt\infty}\,\mathrm{Re}^2_{t\infty}. \qquad [20]$$

From the Fidleris & Whitmore data the wall function can then be easily calculated from [12] as follows:

$$f(\lambda) = \frac{C_{Dt\infty}}{C_{Dt}}\left(\frac{u_{t\infty}}{u_t}\right)^2. \qquad [21]$$

From the above equation, the voidage function for the corresponding multiparticle system is then obtained from [14] and [4]. For one specific Reynolds number the result is shown in figure 7, where it can be seen that the voidage function can again be satisfactorily represented by an expression of the type $\epsilon^{-\beta}$. Values of β have been calculated in this way for a discrete number of Reynolds numbers in the intermediate flow condition and they are depicted in figure 8.

Although the absolute values reported in figure 8 should be used with some caution, due in part to the difficulty in extracting data from the original paper, it is clear that β presents a minimum in the region of Reynolds number between 50 and 100, in complete agreement with the finding for fluid–multiparticle systems. No theoretical explanation is available at the moment justifying the presence of this minimum.

4. THE EXPANSION CHARACTERISTIC OF FLUIDISED BEDS

The simple analogy which has been suggested has proved effective in estimating the magnitude of the voidage function, once the wall function is known. From the voidage function we can then determine the particle–fluid drag force for all the solid–fluid two-phase systems, such as fixed or fluidised beds.

Of particular practical interest, is the determination of the expansion characteristics of homogeneous fluidised suspensions; this expansion law is satisfactorily described by the empirical

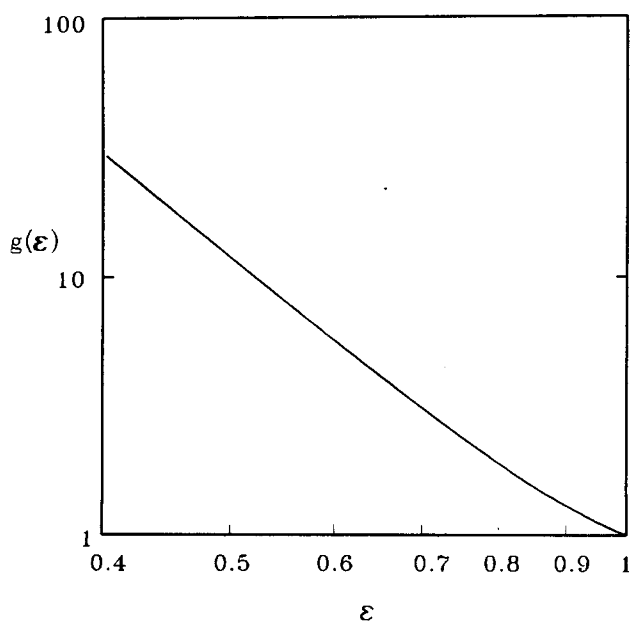

Figure 6. The voidage function evaluated with the present analogy in the inertial flow regime. Line obtained from [19].

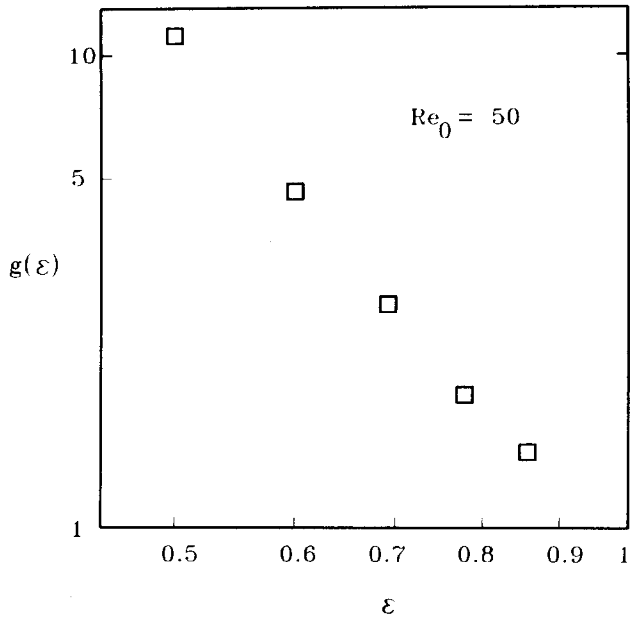

Figure 7. The voidage function evaluated with the present analogy in the intermediate flow regime. Points obtained from the data of Fidleris & Whitmore (1961).

Richardson & Zaki (1954) equation

$$\frac{u_0}{u_{t\infty}} = \epsilon^n. \quad [22]$$

The parameter n is function of the fluid dynamic regime only, and figure 9(a) depicts the expansion characteristics for various systems utilising values of n as recommended in the original Richardson & Zaki (1954) work.

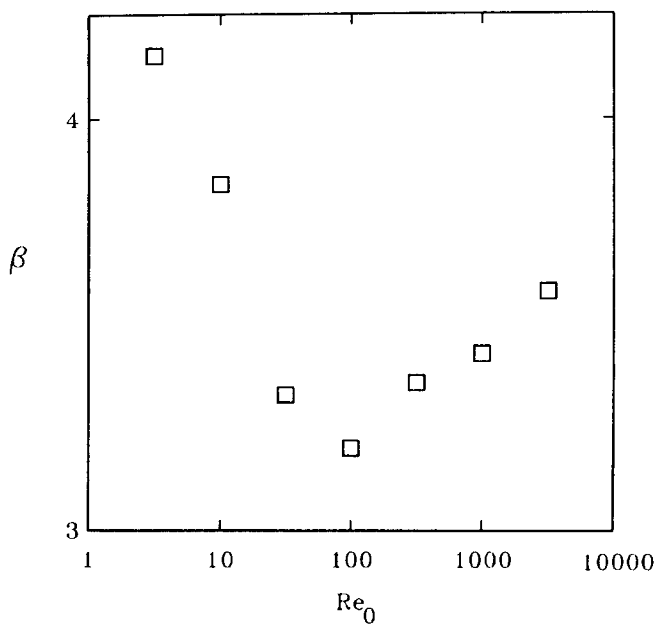

Figure 8. The exponent β evaluated with the present analogy function of the Reynolds number. Points obtained from the data of Fidleris & Whitmore (1961).

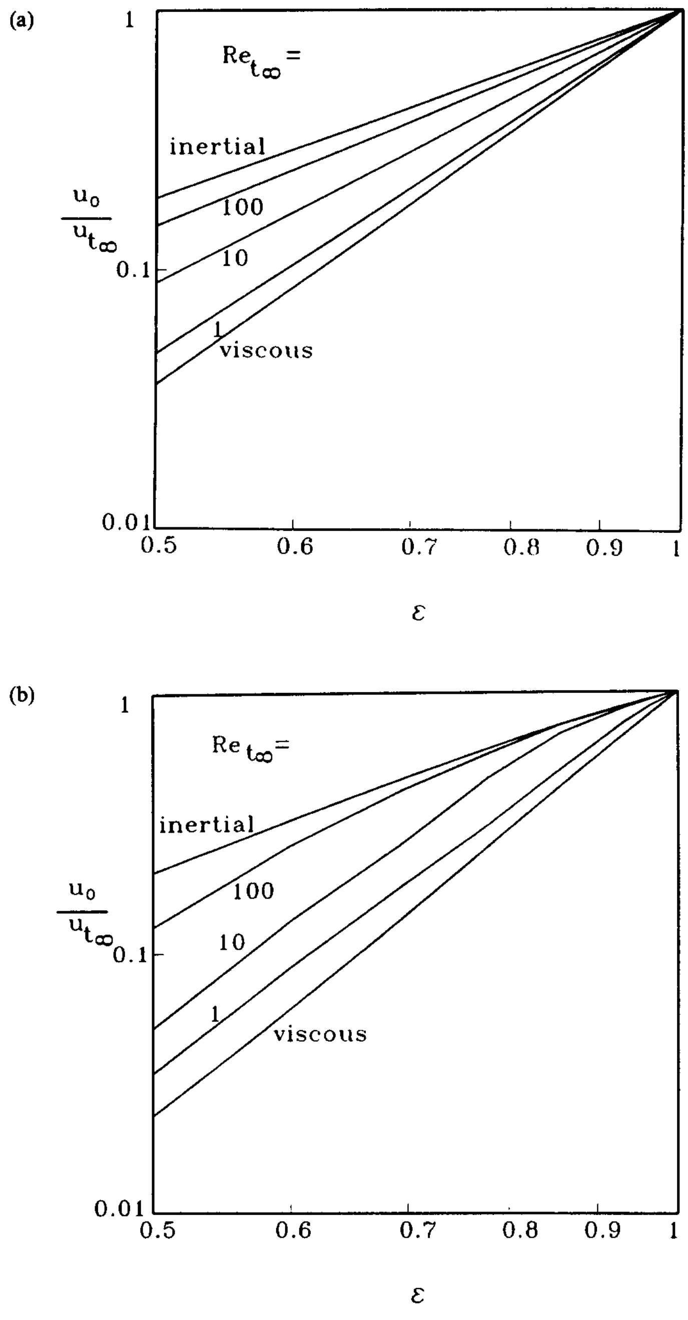

Figure 9. The expansion characteristics of homogeneous fluidised beds. (a) Experimental expansions as reported by Richardson & Zaki (1954); (b) expansions evaluated with the present analogy.

We are in a position now to be able to estimate the expansion of homogeneous fluidised beds. First of all, its relation with the voidage function should be recalled (Di Felice 1994)

$$\frac{u_0}{u_{t\infty}} = \left(\frac{C_{Dt\infty}\epsilon}{C_{D0}g(\epsilon)}\right)^{0.5}. \qquad [23]$$

The introduction of the relationship between wall function and voidage function, [14], leads to

$$\frac{u_0}{u_{t\infty}} = \left(\frac{C_{Dt\infty}\epsilon^3}{C_{Dc}f(\lambda)}\right)^{0.5}. \qquad [24]$$

Again we can obtain simplified expression for the limiting flow regime:

$$\frac{u_0}{u_{t\infty}} = \frac{\epsilon^2}{f(\lambda)} \qquad [25]$$

for the viscous region, and

$$\frac{u_0}{u_{t\infty}} = \left(\frac{\epsilon^3}{f(\lambda)}\right)^{0.5} \qquad [26]$$

for the inertial region.

If, like our case in the intermediate flow regime, the wall function is not known directly, but its magnitude can be inferred from experimental measurements of the retarding effect of the wall on the single particle terminal settling velocity, then a remarkably simple expression is obtained, from [21] and [24]

$$\frac{u_0}{u_{t\infty}} = \frac{u_t}{u_{t\infty}}\epsilon^{1.5}. \qquad [27]$$

Figure 9(b) reports the expansion characteristics of fluidised beds as obtained with the present analogy. For the viscous and inertial regime, available theoretical expressions for the wall function have been used, [17] and [19], respectively, coupled with [25] and [26]. In the intermediate range of terminal Reynolds numbers the experimental findings of Fidleris & Whitmore (1961) have been utilised, coupled with [27].

A quite satisfactory agreement is evident when figure 9(a) and (b) are compared.

5. CONCLUSIONS

The proposed analogy between a single particle in a tube and a multiparticle suspension has proved to be quite successful: expressions for the interaction force in a solid–fluid system have been obtained starting from the knowledge of the effect of the wall on a lone particle in a tube. For the limiting flow regimes, this effect has been established theoretically: as a consequence we have been able to estimate the solid–fluid interaction force without any empirical input.

Acknowledgement—This work was supported by the Ministero dell'Università e della Ricerca Scientifica e Tecnologica, Rome (MURST 40%).

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